■ Examples of Ring:

1. $(\mathbb{Z},+)$ is a commutative group and $(\mathbb{Z},\cdot)$ is a commutative monoid, 1 being the identity element. The distributive law holds. Therefore, $(\mathbb{Z},+,\cdot)$ is a commutative ring with unity.

2. $(\mathbb{Q},+,\cdot)$ is a commutative ring with unity.

3. $(\mathbb{R},+,\cdot)$ is a commutative ring with unity.

4. $(\mathbb{C},+,\cdot)$ is a commutative ring with unity.

5. $(2\mathbb{Z},+)$ is a commutative group and $(2\mathbb{Z},\cdot)$ is a commutative semi-group. The distributive law holds.

Therefore, $(2\mathbb{Z},+,\cdot)$ is a commutative ring. It is a ring with unity.

NOTE Let $n \in \mathbb{N}$. Then $(n\mathbb{Z},+,\cdot)$ is a commutative ring. This is a ring with unity.

6. Ring of real matrices: Let $M_2(\mathbb{R})$ be the set of all 2×2 matrices whose elements are real numbers. $(M_2(\mathbb{R}),+)$ is a commutative, where $+$ denotes matrix addition and $(M_2(\mathbb{R}),\cdot)$ is a monoid, where $\cdot$ denotes matrix multiplication. The distributive laws hold.

Therefore, $(M_2(\mathbb{R}),+,\cdot)$ is a ring with unity.

The identity matrix I_2 is the unity in the ring.

This is a non-commutative ring.

Let, $n \in \mathbb{N}$. Then $(M_n(\mathbb{R}), +, \cdot)$ is the ring of all $n \times n$ real numbers. It is a non-commutative ring with unity, I_n being the unity in the ring.

7. Ring of integers modulo n:

For a fixed $n \in \mathbb{N}$, let $\mathbb{Z}_n$ be the classes of residues of integers modulo n. $\mathbb{Z}_n = \{\bar{0}, \bar{1}, \bar{2}, \ldots, \overline{n-1}\}$.

$(\mathbb{Z}_n, +)$ is a commutative group, where $+$ denotes addition (mod n).

$(\mathbb{Z}_n, \cdot)$ is a commutative monoid where $\cdot$ denotes multiplication (mod n). The distributive law holds.

Therefore, $(\mathbb{Z}_n, +, \cdot)$ is a commutative ring with unity. $\bar{1}$ is the unity in the ring.

8. Ring of Gaussian integers:

Let us consider the subset of $\mathbb{C}$ given by

$\mathbb{Z}[i] = \{a + ib : a, b \in \mathbb{Z}\}$.

$\mathbb{Z}[i]$ is the set of all complex numbers of the form $a + ib$, where a and b are integers.

$\mathbb{Z}[i]$ forms a ring under addition and multiplication of complex numbers. This is a commutative ring with unity.

This ring is called the ring of Gaussian integers.

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